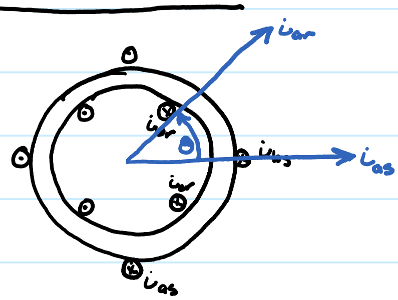
Induction machines:



* Same equations for flux linkage as for synchronous machines

* $\theta = \omega_m t + \gamma$

Recall, for constant torque: $\omega_m = \omega_s - \omega_r$

* For synchronous, we said $\omega_r = 0$.

* For induction, $\omega_r \neq 0$, and introduce slip: $s = \frac{\omega_s - \omega_m}{\omega_s}$ so that $\boxed{\omega_r = s\omega_s}$ slip frequency

Assume:

$i_{ar} = I_r \cos(s\omega_s t + \beta)$ $\quad$ $i_{as} = I_s \cos(\omega_s t)$ $\quad$ $\theta = (1-s)\omega_s t + \gamma$

$i_{br} = I_r \sin(s\omega_s t + \beta)$ $\quad$ $i_{bs} = I_s \sin(\omega_s t)$

Voltage: $V_{as} = i_{as} R_s + \frac{d\lambda_{as}}{dt}$

after simplification: $V_{as} = R_s I_s \cos(\omega_s t) - \omega_s L_s I_s \sin(\omega_s t) - \omega_s M I_r \sin(\omega_s t + \gamma + \beta)$

$$\boxed{\begin{aligned} V_{as} &= R_s I_s \cos(\omega_s t) + \omega_s L_s I_s \cos\left(\omega_s t + \tfrac{\pi}{2}\right) + \omega_s M I_r \cos\left(\omega_s t + \gamma + \beta + \tfrac{\pi}{2}\right) \\ V_{as} &= V_s \cos(\omega_s t + \theta_{pf}) \end{aligned}}$$

$V_{ar} = R_r i_{ar} + \frac{d\lambda_{ar}}{dt}$

$\Rightarrow V_{ar} = R_r I_r \cos(s\omega_s t + \beta) - s\omega_s L_r I_r \sin(s\omega_s t + \beta) - s\omega_s M I_s \sin(s\omega_s t - \gamma)$

$$\boxed{\begin{aligned} V_{ar} &= R_r I_r \cos(s\omega_s t + \beta) + s\omega_s L_r I_r \cos\left(s\omega_s t + \beta + \tfrac{\pi}{2}\right) + s\omega_s M I_s \cos\left(s\omega_s t - \gamma + \tfrac{\pi}{2}\right) \\ V_{ar} &= V_r \cos(s\omega_s t + \theta_r) \end{aligned}}$$

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Phasors:

$$\frac{V_s}{\sqrt{2}}\angle\theta_{PF} = \frac{R_s I_s}{\sqrt{2}}\angle 0 + j\omega_s L_s \frac{I_s}{\sqrt{2}}\angle 0 + j\omega_s M \frac{I_r}{\sqrt{2}}\angle \gamma+\beta$$

$$\frac{V_r}{\sqrt{2}}\angle\theta_r = \frac{R_r I_r}{\sqrt{2}}\angle\beta + js\omega_s L_r \frac{I_r}{\sqrt{2}}\angle\beta + js\omega_s M \frac{I_s}{\sqrt{2}}\angle -\gamma$$

$$\frac{V_r}{\sqrt{2}}\angle\theta_r+\gamma = R_r \frac{I_r}{\sqrt{2}}\angle\gamma+\beta + js\omega_s L_r \frac{I_r}{\sqrt{2}}\angle\gamma+\beta + js\omega_s M \frac{I_s}{\sqrt{2}}\angle 0$$

Recall: $M = \frac{\mu_0 \pi r l N_s N_r}{2g}$

$$L_s = L_{ls} + \left(\frac{N_s}{N_r}\right)M$$

$$L_r = L_{lr} + \left(\frac{N_r}{N_s}\right)M$$

Define rotor values referenced to the stator: $I_r' = I_r\left(\frac{N_r}{N_s}\right)$ $\quad V_r' = V_r\left(\frac{N_s}{N_r}\right)$

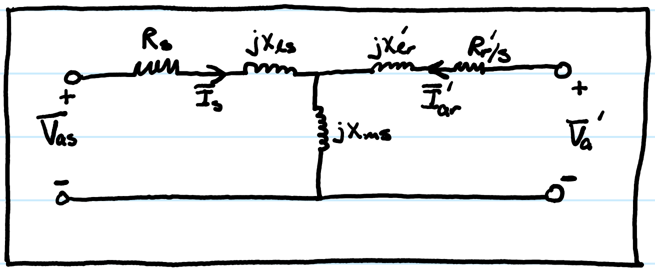
$R_r' = R_r\left(\frac{N_s}{N_r}\right)^2$ $\quad L_{lr}' = L_{lr}\left(\frac{N_s}{N_r}\right)^2$

This then gives:

$$\frac{V_s}{\sqrt{2}}\angle\theta_{PF} = R_s\frac{I_s}{\sqrt{2}}\angle 0 + j\omega_s L_{ls}\frac{I_s}{\sqrt{2}}\angle 0 + j\omega_s\left(\frac{N_s}{N_r}\right)M\frac{I_s}{\sqrt{2}}\angle 0 + j\omega_s\left(\frac{N_s}{N_r}\right)M I_r'\angle\gamma+\beta$$

$$\frac{V_r'}{s\sqrt{2}}\angle\theta_r+\gamma = \frac{R_r'}{s}\frac{I_r'}{\sqrt{2}}\angle\gamma+\beta + j\omega_s L_{lr}'\frac{I_r'}{\sqrt{2}}\angle\gamma+\beta + j\omega_s\left(\frac{N_s}{N_r}\right)M\frac{I_r'}{\sqrt{2}}\angle\gamma+\beta + j\omega_s\left(\frac{N_s}{N_r}\right)M\frac{I_s}{\sqrt{2}}\angle 0$$

$X_{ls} = \omega_s L_{ls}$ $\quad X_{ms} = \omega_s\left(\frac{N_s}{N_r}\right)M$ $\quad X_{lr}' = \omega_s L_{lr}'$

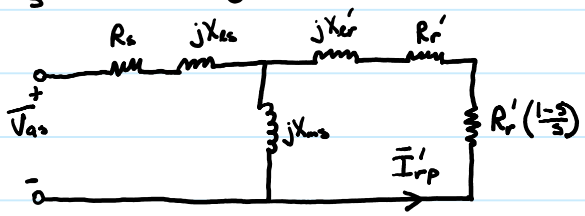


* Two types of machines:

1) wound rotor: connect something to $\overline{V}_r'$
2) Squirrel cage: $V_r' = 0$

Let $V_r' = 0$

$\frac{R_r'}{s} = R_r' + \frac{R_r'(1-s)}{s}$



$* \ \bar{I}_r' \neq \bar{I}_{rp}'$ (different angles)

3phase:

$P_{in} = Re\{3\bar{V}_s \bar{I}_s^*\}$

$P_{SCL} = 3|\bar{I}_s|^2 R_s$ (stator copper loss)

$P_{rCL} = 3|\bar{I}_r'| R_r'$ (rotor copper loss)

$P_m = 3|\bar{I}_r'|^2 R_r'\left(\frac{1-s}{s}\right)$ (mechanical power)

$T^e = \frac{P_m}{\omega_m}$

$P_{shaft} = P_m - P_{rot}$ (P_{rot} is rotational power loss)

$P_{AG} = 3|\bar{I}_r'|^2 \frac{R_r'}{s} = \frac{P_m}{1-s}$ (power across air gap)

$* \ P_m = (1-s)P_{AG} \ *$

For multi-pole systems: $\omega_m = (1-s)\omega_s \frac{2}{P}$ $\qquad \omega_s = 2\pi f$

$T^e = \frac{(1-s)P_{AG}}{(1-s)\omega_s\left(\frac{2}{P}\right)} \Rightarrow T^e = \frac{P_{AG}}{\frac{2}{P}\omega_s}$